# The F Test for Equality of Variance

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# F Test for Equality of Variance - Motivation

- What if instead of testing a hypothesis about a population **mean**, **median**, or **proportion** we want to test a hypothesis about a **variance**?
  - For example, say we have two independent samples and want to know whether their population variances are equal or not

- There is no general method to do this, but in the special case that **both populations are normal** we can use the a method called the **F test**

# The Null Hypotheses

- Let $X_1,\ldots,X_m$ be a random sample from a $N(\mu_1, \sigma^2_1)$ population and let $Y_1,\ldots,Y_n$ be an **independent** sample from a $N(\mu_2, \sigma^2_2)$ population

$$H_0: \sigma^2_1 \leq \sigma^2_2 \quad \text{or} \quad \sigma^2_1/\sigma^2_2 \leq 1$$

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- We will need a test statistic that measures evidence against the null hypothesis
  - Reject two-sided $H_0$ when the ratio of the variances is far from 1 (in either direction)
  - How do we estimate the ratio of the (unknown) population variances?

**$F = s^2_1/s^2_2$ <- this is our test statistic!**

# Why is This Useful?

When would we actually want to test this sort of hypothesis?

- If we reject $H_0$: evidence suggests variances are not equal
- If we do not reject $H_0$: we don't have enough evidence to say that the variances are not equal

Q: Can we use it to decide which version of the t-test to use in a HT of population means (equal or unequal variance)?

A: No! Recall that we can only use the equal variances version if we **know** that the variances are equal

- Not rejecting $H_0$ doesn't mean we are concluding that it is true
- The F test doesn't help us decide!

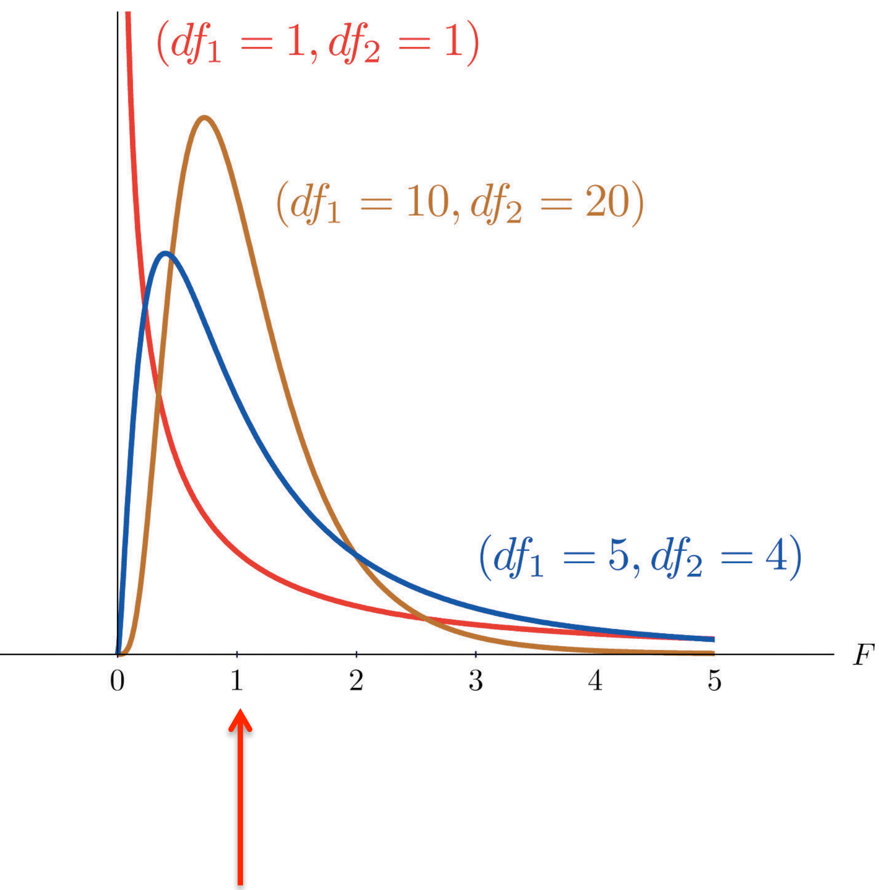
# OK… So Why is This Useful?!

- It is *very* useful in the context of linear regression and analysis of variance (Chapters 8 and 9)

- Essentially, it will help us choose between different **models** for our data by testing which one explains more of the variation

- For now, just focus on learning how to perform the steps of the F test, knowing that you'll need it later

# The F Test Statistic & F Distribution

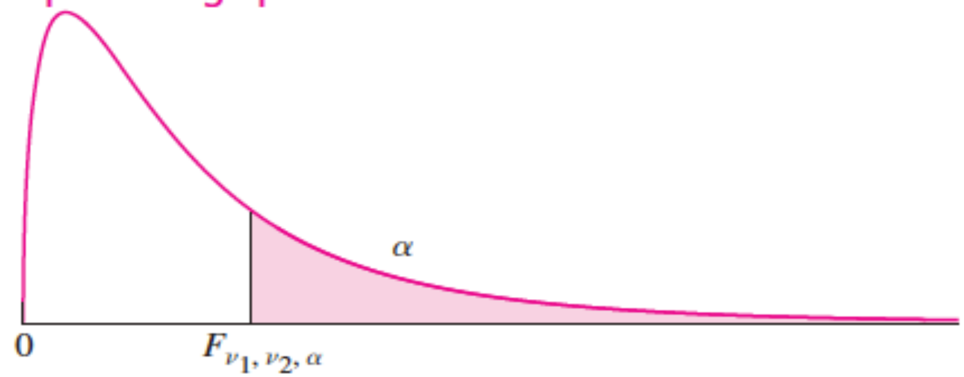



Note the concentration around 1!

- The test statistic is: $F = s^2_1/s^2_2$
- The F statistic will follow the **F distribution** when $H_0$ is true
- The F distribution has **two** degree of freedom parameters $df_1$ and $df_2$ (for numerator & denominator) that determine its shape
- The degrees of freedom relate to the sample sizes m and n (order is important!)
- Under $H_0$: $\mathbf{F \sim F_{m-1, n-1}}$

# How to Use an F Table

**TABLE A.8** Upper percentage points for the $F$ distribution




| | | $\nu_1$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\nu_2$ | $\alpha$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 0.100 | 39.86 | 49.50 | 53.59 | 55.83 | 57.24 | 58.20 | 58.91 | 59.44 | 59.86 |
| | 0.050 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 |
| | 0.010 | 4052.18 | 4999.50 | 5403.35 | 5624.58 | 5763.65 | 5858.99 | 5928.36 | 5981.07 | 6022.47 |
| | 0.001 | 405284 | 500012 | 540382 | 562501 | 576405 | 585938 | 592874 | 598144 | 603040 |
| 2 | 0.100 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.38 |
| | 0.050 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 |
| | 0.010 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 |
| | 0.001 | 998.50 | 999.00 | 999.17 | 999.25 | 999.30 | 999.33 | 999.36 | 999.37 | 999.39 |
| 3 | 0.100 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.24 |
| | 0.050 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 |
| | 0.010 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 |
| | 0.001 | 167.03 | 148.50 | 141.11 | 137.10 | 134.58 | 132.85 | 131.58 | 130.62 | 129.86 |

# Summary: F Test

Let $X_1,\dots,X_m$ be a random sample from a $N(\mu_1, \sigma^2_1)$ population and let $Y_1,\dots,Y_n$ be an **independent** sample from a $N(\mu_2, \sigma^2_2)$ population where $s^2_1 > s^2_2$.

1. Set up the null and alternative hypotheses (see table below)
2. State the level of significance α
3. Calculate the test statistic: $F = s^2_1/s^2_2 \sim F_{m-1,\, n-1}$ or
   $F^* = s^2_2/s^2_1 \sim F_{n-1,\, m-1}$
4. Find the p-value using the F distribution

| $H_0$ | $H_1$ | P-value |
|---|---|---|
| $\sigma^2_1/\sigma^2_2 \leq 1$ | $\sigma^2_1/\sigma^2_2 > 1$ | Area to the right of F |
| $\sigma^2_1/\sigma^2_2 \geq 1$ | $\sigma^2_1/\sigma^2_2 < 1$ | Area to the right of F* |
| $\sigma^2_1/\sigma^2_2 = 1$ | $\sigma^2_1/\sigma^2_2 \neq 1$ | 2*(Area to the right of F) |

5. Make a conclusion based on the p-value

Why? $\sigma^2_1/\sigma^2_2 < 1$ is equivalent to $\sigma^2_2/\sigma^2_1 > 1$ and the table only provides right-tails

# Example 6.23 – Variance of Pesticide Absorption

- We are interested in the variances of the skin absorption rates for two different pesticides A and B
- An experiment was carried out in which the amount of absorbed pesticide (μg) after a certain time period was measured
- **Pesticide A**: sample variance was 2.3 $\mu g^2$ for a sample of size 6
- **Pesticide B**: sample variance was 0.6 $\mu g^2$ for a sample of size 10
- Assume the samples are independent and that the absorption rates are normally distributed
- Can we conclude that the variance in the amount absorbed is greater for pesticide A than for pesticide B?

# Warnings

- The F test is fairly **sensitive** to the assumption that the samples come from normal populations
  - No large-sample property comes to our rescue here!

- If the shapes of the populations differ much from the normal curve, the F test may give misleading results

- The F test does **not** prove that two variances are equal

# Next

- Type I and Type II errors
- Power
- Multiple Testing Issues